

16.61 - Aerospace Dynamics Spring 2006

Homework 2

Version: 1.0

Prof. Charles P. Coleman

Date Out: Friday 17 February 2006

Date Due: Friday 24 February 2006 2pm

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	

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Problem 1

Two right-handed orthogonal frames A and B are rotating with respect to (w.r.t.) each other. Frame B is rotating w.r.t frame A at angular rate $\omega^{B/A}$. Accordingly, frame A is rotating w.r.t. frame B at angular rate $\omega^{A/B}$. Use Coriolis' Theorem to show that the angular acceleration, $\dot{\omega}$, can be measured in either frame. That is, show

$$\dot{\omega} = \dot{\omega}^{B/A} = \dot{\omega}^{A/B}$$

Reference:

[1] Wie, Space Vehicle Dynamics and Control, Problem 1.6 (a), p 27, AIAA, 1998.

Problem 2

Prove the following statement:

$$S(R\omega) = RS(\omega)R^T$$

Give some insight as to what this statement means?

Reference:

[2] Spong & Vidyasagar, Robot Dynamics and Control, Example 2.5.2 and Problem 2-22, pp 51–52, 59, John Wiley, 1989.

Problem 3

It is often important to find the resultant or absolute angular velocity due to the relative rotation of several coordinate frame. Show that the angular velocity of frame B with respect to frame A can be expressed as follows

$$\omega^{B/A} = \omega^{B/A_1} + \omega^{A_1/A_2} + \omega^{A_2/A_3} + \dots + \omega^{A_n/A}$$

where A_1, \dots, A_n are n auxiliary reference frames.

References:

[1] Wie, Space Vehicle Dynamics and Control, Problem 1.6 (a), p 27, AIAA, 1998.

[2] Spong & Vidyasagar, Robot Dynamics and Control, Section 2.7, pp 55–56, John Wiley, 1989.

Problem 4

Three frames A , B , and C are given below. If the angular velocities $\omega^{B/A}$ and $\omega^{C/B}$ are given as

$$\omega^{B/A} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} ; \quad \omega^{C/B} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

what is the angular velocity $\omega^{B/A}$ at the instant when

$$R_A^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

Reference:

[2] Spong & Vidyasagar, Robot Dynamics and Control, Problem 2-34, p 61, John Wiley, 1989.

Problem 5

Two frames A and B are related by the homogenous transformation

$$H_A^B = \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A particle has velocity $v_B(t) = (3, 1, 0)^T$ relative to frame B . What is the velocity of the particle in frame A ?

Reference:

[2] Spong & Vidyasagar, Robot Dynamics and Control, Problem 2-33, p 61, John Wiley, 1989.